# *Logic Programs as Compact Denotations*

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# *Credits*

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EPSRC grant “Escape Analysis of Object-Oriented Languages”

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- Compositionality is obtained by using a denotational semantics
- Denotations are maps. We show how to compact them and make them more efficient
- This is achieved through their representation in terms of logic programs
- Some experiments show the advantages of using our technique even compared to BDD's.

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- compositional: $[\![c_1; c_2]\!] = [\![c_1]\!] \circ [\![c_2]\!]$
- easy to implement (no memoization)

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- it's an input/output map!
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- it can be concrete or abstract
- if the abstract domain is finite, also the abstract map $[\![c]\!]$ is finite for every command $c$.

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$$d(\mathtt{empty}) = \mathtt{empty}$$

$$d([+,+,+,+]) = [+,+,+,+] \quad d([+,+,+,-]) = [+,+,+,-]$$

$$d([+,+,-,+]) = [+,+,-,+] \quad d([+,+,-,\mathtt{u}]) = [+,+,-,\mathtt{u}]$$

$$d([+,-,+,+]) = [-,-,+,+] \quad d([+,-,\mathtt{u},-]) = [-,-,\mathtt{u},-]$$

$$d([+,-,-,+]) = [-,-,-,+] \qquad \ldots$$

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  ? we use it to handle recursion
  $\Rightarrow$ we need a check for function equality to stop the computation.

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$\Downarrow$

Computing the abstract denotational semantics of a possible very small program looks computationally expensive.

# *Small Improvements*

$$d([+,+,+,+]) = [+,+,+,+]$$
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Very fast and compact in practice.

Why? Because many functions contain regularity or default cases that get exploited by the BDD's normal form.

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3. Graphs can be kept in normal (minimal) form
4. Operations over graphs correspond to $\circ$, $\cup$ and functional equality.

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  $\Rightarrow$ we must find the encoding and program the encoder and the decoder (to show the output to the user).

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$$d([\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{O}]) = [\mathtt{B}, \mathtt{B}, \mathtt{C}, \mathtt{O}]$$

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O(n) !!

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Only facts are used.

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- Type analysis of ML
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No general theory. No application to imperative programs

# *Compact Denotations*

A compact denotation is a possibly non-ground set of arrows

$$cd = \{L_1 \to r_1, \ldots, L_m \to r_m\}$$

$$cd(L_1) = r_1$$

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Its meaning is the denotation $\overline{cd}$ obtained by grounding the variables *w.r.t.* a set of legal substitutions.

# *Constraints on Compact Denotations*



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We require that compact denotations:

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- have non overlapping heads
- have monotonic meaning.

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$$t_1 \circ^* t_2 = \left\{ (l_2 \to r_1)\theta \;\middle|\; \begin{array}{l} l_2 \to r_2 \in t_2,\ l_1 \to r_1 \in t_1 \\ \texttt{domain\_entails}(l_1, r_2) \\ \text{computes } \theta \end{array} \right\}.$$

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`domain_entails` is domain-dependent!

# An Example

$$\texttt{empty} \rightarrow \texttt{empty}$$
$$[\texttt{B}, +] \rightarrow [-]$$
$$[\texttt{B}, -] \rightarrow [+]$$

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$$\begin{array}{lcl} \texttt{empty} \to \texttt{empty} & & \texttt{empty} \to \texttt{empty} \\ [\texttt{B}, +] \to [-] & \circ^* & [\texttt{A}, +] \to [-, \texttt{u}] \\ [\texttt{B}, -] \to [+] & & [\texttt{A}, -] \to [+, +] \end{array}$$

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This is not a compact denotation!

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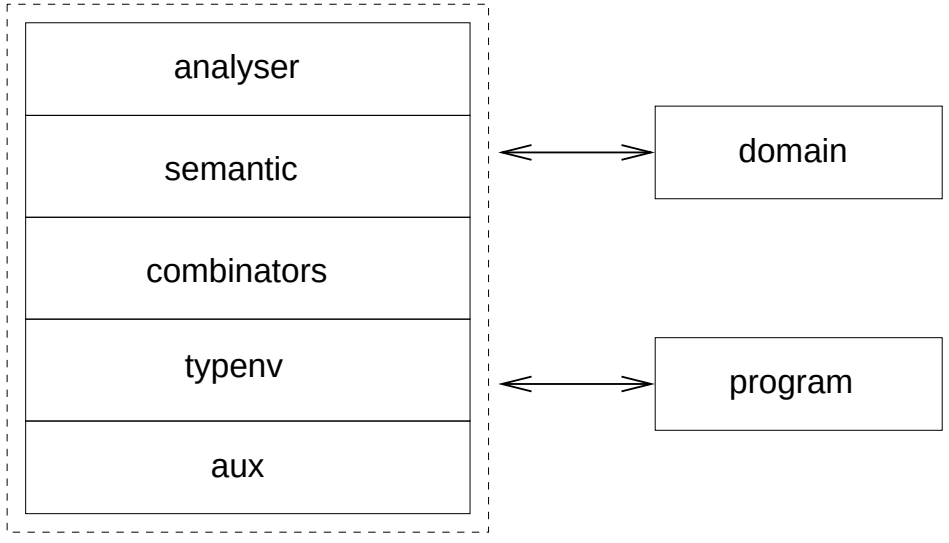
- $\cup$ can be implemented in terms of $\circ^*$ and the lub of the abstract domain [Spoto, SAS 2001]
- for every basic operation of the language we have provided basic compact denotations
- equality on compact denotations is first checked syntactically and then semantically.

# *LOOP*

The Localised Analyser for Object-Oriented Programs...




www.sci.univr.it/∼spoto/loop

# *Our Abstract Domains*

| analysis | ground | non-ground | BDD's |
|---|---|---|---|
| sign | `signs` | `signs_vars` | `signs_bdd` |
| rapid type | `rt` | `rt_vars` | |
| dataflow class | `df` | `df_vars` | |
| P&S class | `ps` | `ps_vars` | |
| escape | `e` | `e_vars` | |

# *Sign Analysis*

| Bmrk | signs | | signs_vars | |
|---|---|---|---|---|
| | time | space | time | space |
| arith | 108.578 | 13617292 | 0.163 | 26242 |

# *Rapid Type Analysis*

| Bmrk | rt | | rt_vars | |
|---|---|---|---|---|
| | time | space | time | space |
| figures | 2.956 | 776338 | 1.193 | 295880 |

# *Dataflow Class Analysis*

| Bmrk | df | | df_vars | |
|---|---|---|---|---|
| | time | space | time | space |
| clone | 11.450 | 2320420 | 0.078 | 18935 |

# *P&S Class Analysis*

| Bmrk | ps | | ps_vars | |
|---|---|---|---|---|
| | time | space | time | space |
| clone | 284.624 | 29393040 | 0.165 | 47855 |

# *Escape Analysis*

| Bmrk | e | | e_vars | |
|---|---|---|---|---|
| | time | space | time | space |
| inv | 0.432 | 14060 | 0.217 | 9268 |

# *Comparison with BDD's*

| Bmrk | signs_vars | | signs_bdd *small* | |
|---|---|---|---|---|
| | time | space | time | space |
| arith | 0.198 | 38485 | 0.481 | 1000 |

| Bmrk | signs_vars | | signs_bdd *large* | |
|---|---|---|---|---|
| | time | space | time | space |
| arith | 0.198 | 38485 | 0.168 | 1000000 |

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- . . . which is more complicated to write!

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- They are less effective for the domains which do not use a per-variable approximation (rapid type and escape analyses).

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# Q & A